Stretching and Folding in the KIII Neurodynamical Model

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Abstract

In this paper we provide an alternate view of the KIII model derived from the laws of complexity (Stretching and Folding) rather than the laws of physics. This approach requires the derivation of Infinitesimal Diffeomorphisms (ID) in place of Ordinary Differential Equations. We indicate how IDs originate and then use them to replicate several examples from the work of Freeman and Kozma. By viewing the KIII theory as a pure mathematical system we anticipate that the KIII Theory will be made more accessible to researchers and scientists unfamiliar with the details of neuroscience and thus offer advances to the KIII Theory from other perspectives.

1 Introduction

Freeman and Kozma have introduced a paradigm shift in the analysis of neurodynamics by focusing on the mesoscopic structures external to the neurons referred to as the neuropil [1], rather than the dynamics of the neuron mass only.

The central question that any neurodynamical theory must answer is this: how is intentionality communicated from the limbic system (the seat of intentionally) to specific regions of the brain in a manner that causes those regions to arrange or configure themselves to perform a new desired task? For example, how does one learn to hit a tennis ball given that they have never picked up a tennis racquet? The range of dynamical activities involved in leaning a complex task which must be performed in a few seconds and which involves thought, action and emotion, for the first time, crosses the entire spectrum of human capability. Since mitosis is not operable within most of the neural mass, the problem of communicating intent and rapidly learning a new task driven by intent is difficult to explain in a neurocentric theory. Thus, a new communication and learning paradigm is essential to address these questions.

An understanding of the amorphous nature of the neuropil, more analogous to a stiff fluid or a shag rug, suggested an entirely new approach to neurodynamical modeling that uses a field or wave paradigm as the means of communication, and the neuropil as the medium over which these waves must travel to relevant regions of the brain. It is on this fundamental wave-based neuropil approach that the KIII model is built.

In this article we shall refer to the KIII model as a bottoms-up approach. Using a bottoms-up approach, the laws of physics are applied to derive a system of differential equations known as the KIII model, Eq.(8). We will reverse this approach and use their work as a springboard to develop a top-down model that introduces an entirely new set of laws, the "laws of complexity", with which to derive their theory. It is hoped that the combined bottom-up physics approach and the top-down complexity approach will merge to produce an even greater methodology with which to analyze and perhaps prove theories of neurodynamics; and, through using the top-down approach, we hope to make research of the KIII theory available to a wide range of scientists and mathematicians who do not have an extensive background in neurodynamics.

Our reformulation of KIII is achieved by by applying the fundamental concept of complexity [4], the stretching and folding paradigm of Smale. We introduce the concept of infinitesimal diffeomorphism (ID) as seen in [5] as the mathematical expression of the basic laws of stretching and folding.

2 Stretching and Folding Provide an Alternative Approach to the Laws of Physics for Modeling Dynamics

Whereas the thermodynamical KIII model derives ODEs from the known dynamics of fluids moving across membranes, the ID model begins with identification of the stretching and folding components. From [3] we know that any diffeomorphism of the form

$$\mathbf{X} \to F(\mathbf{X})$$

where $\nabla\nabla \cdot F(\mathbf{X}) \neq 0$ is a stretching component; and any diffeomorphism of the form

$$\mathbf{X} \to \exp(A) \cdot \mathbf{X}$$

where A is a nXn matrix of constants is a folding component.

Using the concept of stretching and folding leads to the concept of Infinitesimal Diffeomorphisms [3] since in the natural world stretching and folding are blended into a single dynamic. To separate these two dynamics it is necessary to identify each individual component. However, due to the continuous nature of the blending of stretching and folding in any dynamic, any model formulated to approximate the dynamics must also blend stretching and folding in very small "infinitesimal" increments. In order to see how to do this ordinary differential equations describing stretching and folding dynamics must be converted to integral equations.

3 The Infinitesimal Diffeomorphisms Originate from Integral Equations

The origin of IDs comes from converting Eq. (8) to an integral equation and then simplifying. An intuitive derivation goes as follows:

$$\begin{aligned}
\exp(-A\cdot(t+h))X(t+h) &= \exp(-A\cdot t)X(t)+\int_t^{t+h}\exp(-As)F(X(s))ds\\
X(t+h)=\exp(A\cdot h)X(t) &+ \exp(A\cdot(t+h))\int_t^{t+h}\exp(-As)F(X(s))ds\\
X(t+h)=\exp(A\cdot h)X(t) &+ \exp(A\cdot(t+h))\int_t^{t+h}(-A^{-1})F(X(s))d\exp(-As)\\
X(t+h)=\exp(A\cdot h)X(t) &+ \exp(A\cdot(t+h))((-A^{-1})F(X(\xi))(\exp(-A(t+h)-\exp(At))\\
X(t+h)=\exp(A\cdot h)X(t) &+ (-A^{-1})F(X(\xi)(1-\exp(A\cdot h))\\
X(t+h)=\exp(A\cdot h)(X(t) &+ (A^{-1})F(X(\xi))(-A^{-1})F(X(\xi))\\
X(t_{n+1}) &\approx \exp(A\cdot h)(X(t_n)-G(X(t_n))+G(X(t_n))
\end{aligned}$$

where $G(X(t))=(-A^{-1})F(X(\xi))$. This requires that A^{-1} exists. When the solution is an attractor, and F is bounded, the ID provides a very good approximation to the solution of a nonlinear autonomous ODE.

Using this form of the ID justifies looking for solutions to any equation of the form Eq.(8) by assuming it has the form of an ID. Our primary guide for applying this approach is partly formal, partly experimental. In general, we start with the form of an ID if we are working with an equation of the form of Eq.(8) and then we use formal data to obtain the best approximation to the stretching terms and folding terms separately. The folding terms will be captured in the eigenvalues of A and the stretching terms will be determined by the form of F and its ”stretching” parameters. The model derived by shifting our emphasis from KIII ODEs to KIII ID will be referred to as KIII-ID.

From an engineering point-of-view, since we are starting with the known form of the solution, using the KIII-ID the numerical approximation and modeling should be achieved with a significant reduction in computational effort. This may come in the form of a reduction in the number of equations needed to model neurodynamics.

3.1 The Linear ID Provides Fundamental Insights into the Dynamics of Stretching and Folding Systems

A linear Infinitesimal Diffeomorphism (ID) is

$$\mathbf{T}_h(\mathbf{X})=\exp(\mathbf{A}\cdot h)(\mathbf{X}-F(\mathbf{X}))+F(\mathbf{X}) \tag{1}$$

where $\mathbf{A} \neq 0$ is an n by n matrix of constants, $\mathbf{X}$ is an n-vector and $\nabla(\nabla \bullet F) \neq 0$, where F is twice differentiable function on $\mathbf{R}^n$ and $h \neq 0$.

The condition, $\nabla(\nabla \bullet F) \neq 0$ is the definition of stretching.

A linear ID inherently combines stretching and folding infinitesimally through the step size h. The folding part is given by $\exp(\mathbf{A} \cdot h)(\mathbf{X})$ since $\nabla(\nabla \bullet (\exp(\mathbf{A} \cdot h)(\mathbf{X})) = 0$ and the stretching part is suppled by F by the condition $\nabla(\nabla \bullet F) \neq 0$.

Consider

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + F(\mathbf{X}) \tag{2}$$

If F is bounded, then Eq.(1) accurately approximates the solution of Eq.(2):

Theorem 1 *Let F if bounded on $\mathbf{R}^n$ then for h_1, h_2, then*

$$\|\mathbf{T}_{h_1}(\mathbf{X}) - \mathbf{T}_{h_2}(\mathbf{X})\| \leq K\|h_1 - h_2\|\|\mathbf{X}\| \tag{3}$$

for some constant K which depends on the bound of F.

The fixed points of $\mathbf{T}$ are given by

$$\mathbf{T}_h(\mathbf{X}) = \mathbf{X} \tag{4}$$

or

$$\exp(\mathbf{A} \cdot h)(\mathbf{X} - F(\mathbf{X})) = \mathbf{X} - F(\mathbf{X}) \tag{5}$$

For nonzero h, Eq. (5) implies $\mathbf{X} - F(\mathbf{X})$ belongs to the kernel of $\mathbf{A}$. Thus set of fixed points of the one-parameter family $\mathbf{T}_h$ is precisely the kernel of $\mathbf{A}$. Some of the fixed points of the linear ID are given by $F(\mathbf{X}) = \mathbf{X}$. The dynamics of the fixed points are given by the Jacobian of $\mathbf{T}$.

3.2 The Nonlinear ID provides a General Approach to Modeling Dynamics

The nonlinear case is designated by allowing $\mathbf{A}$ to be a matrix function of $\mathbf{X}$:

$$\mathbf{T}_h(\mathbf{X}) = \exp(\mathbf{A}(\mathbf{X})) \cdot h)(\mathbf{X} - F(\mathbf{X})) + F(\mathbf{X}) \tag{6}$$

where $\mathbf{A}(\mathbf{X})$ is function of $\mathbf{X}$ and as in the liner case, $\nabla(\nabla \bullet F) \neq 0$.

Some of the fixed points in the nonlinear case correspond to the zeros of $\mathbf{A}(\mathbf{X})$.

The synchronous nonlinear case is when $\mathbf{A}(\mathbf{X}) = \mathbf{A}(F(\mathbf{X}))$. For fixed h the set of all synchronous ID is designated by $\mathscr{S}(h)$.

This implies, generally, that the real part of the eigenvalues of the fixed points of $\mathbf{T}$ vary with the center of the diffeomorphism defined by $F(\mathbf{X})$

For fixed h, the set of all bounded complex valued continuous functions, f on $\mathbf{R}^n$ induces an algebra on $\mathscr{S}(h)$ as follows: Let $\mathbf{T}(f), \mathbf{T}(g) \in \mathscr{S}(h)$ then $\mathbf{T}(f) \oplus \mathbf{T}(g) = \mathbf{T}(f+g)$ etc. Also we have

$$\mathbf{T}^*(f) = \mathbf{T}(\bar{f})$$

Some of the fixed points for the set of synchronous IDs are determined by the zeros of F. When $F(\mathbf{X}) = 0$ the synchronous ID fixed point equation reduces to

$$\mathbf{T}_h(\mathbf{X}) = \mathbf{X} = \mathbf{I}(\mathbf{X}) \tag{7}$$

since $\exp(0) = \mathbf{I}$

In the nonlinear case, the ID remains a good approximation of the time series for the associated nonlinear ODE which, typically, cannot be derived in closed form.

The simple scroll is an example of a nonlinear synchronous ID [5].

3.3 The Standard KIII model can be Reformulated as a Set of Infinitesimal Diffeomorphisms (ID)

The standard thermodynamic KIII model can be described by a vector equation whose most general form is Eq.(8). Note that in[1] second order ODEs are used as a basis for formulating te KIII model. To translate this into IDs, we replace each second order ODE with a pair of IDs.

$$\frac{d\mathbf{X}}{dt} = \mathbf{AX} + F(\mathbf{X}, t) \tag{8}$$

The function $F(\mathbf{X}, t)$ is given as follows, see Eq.8 of [2]:

$$F(\mathbf{X}, t) = \sum_{j}^{N} w_j Q(y_j) + \sum_{j}^{N} \sum_{\tau}^{T} k_{ijk} Q(y_j(t - \tau)) + P_i(t) \tag{9}$$

Eq.(8) can be solved by iterating the vector mapping

$$\mathbf{X}_{n+1} = \exp(\mathbf{A} \cdot h)(\mathbf{X}_n - F(\mathbf{X}_n, t_n)) + F(\mathbf{X}_n, t_n) \tag{10}$$

and when the matrix $\mathbf{A}$ has some eigenvalues less than 1, this approximation can be extremely accurate, see [3], and the step size h can be as large as 0.5 while retaining the morphological properties of the exact solution, [4]. Note that in Eq.(10) the time variable may be absorbed to make the equation ”autonomous” by increasing the dimension by 1.

Equation (10) is required to have sufficiently smooth derivatives. We rewrite Eq.(10) in the form of a transformation on a manifold:

$$\mathbf{T}_h(\mathbf{X}) = \exp(\mathbf{A}\cdot h)(\mathbf{X}_n - F(\mathbf{X}_n)) + F(\mathbf{X}_n) \tag{11}$$

The ID, Eq.(11), has broad applicability and occurs in a wide range of problems of physics, fluid flow and electronic circuits [4].

More generally, an Infinitesimal Diffeomorphism (ID) is a one-parameter family of maps on $\mathbf{R}^n$ of the form (12) where F is a twice differentiable mapping from $\mathbf{R}^n$ to $\mathbf{R}^n$, $G(X)$ is a twice differentiable matrix function of $X \in \mathbf{R}^n$ and $h \neq 0$ is a real parameter.

$$\mathbf{T}_h(\mathbf{X}) = \exp(G(\mathbf{X})\cdot h)(\mathbf{X} - F(\mathbf{X})) + F(\mathbf{X}) \tag{12}$$

As noted previously, the significance of IDs is that they are diffeomorphisms that also have the characteristics of a time series. This fact makes it possible to analyze very complex nonlinear processes more efficiently than by using conventional numerical methods. In addition to the ability to analyze fundamental dynamics, the ID provides an avenue for compression of high-dimensional systems of ODEs due to its similarity to Gaussian integration for second-order ODEs. IDs are particularly well suited to analyze the morphology of nonlinear ODEs of the form (8) which includes such equations as the Chua double scroll, the Lorenz system, the Rössler system and the K-neurodynamical models that will be discussed in this paper.

4 The Application of IDs to K-neurodynamics may Result in Useful Simplifications of the ODEs Use to Describe the KIII System

In this section will apply IDs to formulate the K-neurodynamical models. These models will be designated as the K-ID models.

the K0-ID infinitesimal diffeomorphism is a direct translation of the K0 Eq.1 model in[1]. As noted earlier, this translations replaces a single second order ODE with a pair of IDs.

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} \exp(\alpha\cdot h)\cdot((x_n - F(x_n,y_n))\cdot\cos(\omega\cdot h) + (y_n - F(x_n,y_n))\cdot\sin(\omega\cdot h)) + F(x_n,y_n) \\ \exp(\alpha\cdot h)\cdot((y_n - F(x_n,y_n))\cdot\cos(\omega\cdot h) - (x_n - F(x_n,y_n))\cdot\sin(\omega\cdot h)) + F(x_n,y_n) \end{pmatrix} \tag{13}$$

The KI-ID infinitesimal diffeomorphism is a modified version of the KI model, Eq.3 model in [1]

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \\ w_{n+1} \end{pmatrix} = \begin{pmatrix} \exp(\alpha \cdot h) \cdot ((x_n - F(z_n, w_n)) \cdot \cos(\omega \cdot h) + (y_n - F(z_n, w_n)) \cdot \sin(\omega \cdot h)) + F(z_n, w_n) \\ \exp(\alpha \cdot h) \cdot ((y_n - F(z_n, w_n)) \cdot \cos(\omega \cdot h) - (x_n - F(z_n, w_n)) \cdot \sin(\omega \cdot h)) + F(z_n, w_n) \\ \exp(\alpha \cdot h) \cdot ((z_n - F(x_n, y_n)) \cdot \cos(\omega \cdot h) + (w_n - F(x_n, y_n)) \cdot \sin(\omega \cdot h)) + F(x_n, y_n) \\ \exp(\alpha \cdot h) \cdot ((w_n - F(x_n, y_n)) \cdot \cos(\omega \cdot h) - (z_n - F(x_n, y_n)) \cdot \sin(\omega \cdot h)) + F(x_n, y_n) \end{pmatrix} \tag{14}$$

Rewriting the above equations in the terminology of [1] we have

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \\ w_{n+1} \end{pmatrix} = \begin{pmatrix} \exp(\alpha \cdot h) \cdot ((x_n - Q(v)) \cdot \cos(\omega \cdot h) + (y_n - Q(v)) \cdot \sin(\omega \cdot h)) + Q(v) \\ \exp(\alpha \cdot h) \cdot ((y_n - Q(v)) \cdot \cos(\omega \cdot h) - (x_n - Q(v)) \cdot \sin(\omega \cdot h)) + Q(v) \\ \exp(\alpha \cdot h) \cdot ((z_n - Q(u)) \cdot \cos(\omega \cdot h) + (w_n - Q(u)) \cdot \sin(\omega \cdot h)) + Q(u) \\ \exp(\alpha \cdot h) \cdot ((w_n - Q(u)) \cdot \cos(\omega \cdot h) - (z_n - Q(u)) \cdot \sin(\omega \cdot h)) + Q(u) \end{pmatrix} \tag{15}$$

where

$$Q(s) = q_m \cdot \left(1 - \exp(\frac{1 - \exp(s)}{q_m})\right) \tag{16}$$

and $v = x - 5.23 \cdot w \cdot z$ and $u = y - 0.1 \cdot x$ and $q_m = 5.0$

Again, we see that two second order ODEs are replaced by 4 IDs.

5 The KIII-ID Model can Provide a Reduction in Computation as well as Insights into the Neurodynamics

We now define the KIII-ID as follows:

Assume Eq.(17) is true.

$$\sum_{j}^{N} \sum_{\tau}^{T} k_{ijk} Q(y_j(t - \tau)) \approx Q(f(y_1, y_2, \cdots y_N)) = \Phi(\mathbf{X}) \tag{17}$$

and let $\mathscr{Q}$ be defined as follows:

$$\mathscr{Q} = \sum_{j}^{N} w_j Q(y_j) \tag{18}$$

Let $\Psi(\mathbf{X}) = \mathscr{Q}(\mathbf{X}) + Q(f(y_1, y_2, \cdots y_N) = \mathscr{Q}(\mathbf{X}) + \Phi(\mathbf{X})$

Then KIII-ID is given by

$$\mathbf{T}_h(\mathbf{X}) = \exp(\mathbf{A} \cdot h)(\mathbf{X} - \Psi(\mathbf{X})) + \Psi(\mathbf{X}) \tag{19}$$

The mesoscopic theory requires a wave to pulse dynamic to communicate intent to local regions of the brain responsible for initiating action quickly. This wave dynamic may be what is referred to as a calcium wave in [11] that moves through the neuropil. This leads us to conjecture that the KIII-ID model can be further abstracted by the introduction of a wave/field concept. To arrive at the KIII-ID field model we make the following assumptions:

1. The KIII model was derived from the Neurodynamics of the brain using the simplest possible approach that captures the essential features of EEG studies.

2. The actual dynamics of the brain are so complex that it is reasonable to try to abstract from the KIII model only the essential concepts and dynamics inherent in that model assuming a field theory of the brain.

3. Then derive an abstract model of KIII by reverse engineering the KIII ODE model. To do this, two modification to the KIII theory were introduced:. A) in place of ODEs we used IDs that provide a dramatic simplification of the Runge-Kutta integration approach while retaining all dynamics and providing for step size variation without any loss of morphological accuracy. B) Assume that the forcing function of KIII model, which was derived by direct experimentation, must be morphologically equivalent to a field force having a much simpler form.

4. $\Psi(f(\mathbf{X}))$ is the Field-composite of all interactions between nodes of the KII model. In terms of ID theory, f will represent the transition surface in n-dimensional space which governs the stretching wave in the neuropil. $\exp(\mathbf{A}\cdot h)$ will provide the folding wave component.

Given these abstractions, we present a simulation of the KII-ID model and the KI-ID model and contrast their morphology with EEG recordings from [3]. The simulations are an abstraction of Eq.(13) and Eq.(3) from [1].

FIGURE 1

Data for Fg.(1), KII-ID, are as follows:

Damping and frequency	$\alpha = -0.08 : \beta = 0.9$
Step size	$h = 0.001$
Number of iterations	N = 1000000
Initial conditions	$x = 0 : y = 1 : z = 0 : w = 1.5$
Initial conditions	$x_1 = 0 : y_1 = 1 : z_1 = 1 : w_1 = 1.5$

Iteration equations for KII-ID are as follows:

$$
\begin{aligned}
Q_0 &= Q(y_1) + 0.6\cdot Q(z)\\
Q_1 &= Q(y_1 + w_1) - Q(z)\\
Q_3 &= Q(y - w) + 1.1\cdot Q(z)\\
Q_4 &= Q(y - x)\\
Q(v) &= 5.0\cdot(1 - \exp((1 - \exp(v))/5.0))\\
x &\to \exp(\alpha\cdot h)\cdot((x - Q_1)\cdot\cos(\beta\cdot h) + (y - Q_1)\cdot\sin(\beta\cdot h)) + Q_1\\
y &\to \exp(\alpha\cdot h)\cdot((y - Q_1)\cdot\cos(\beta\cdot h) - (x - Q_1)\cdot\sin(\beta\cdot h)) + Q_1\\
z &\to \exp(\alpha\cdot h)\cdot((z - Q_0)\cdot\cos(\beta\cdot h) + (w - Q_0)\cdot\sin(\beta\cdot h)) + Q_0\\
w &\to \exp(\alpha\cdot h)\cdot((w - Q_0)\cdot\cos(\beta\cdot h) - (z - Q_0)\cdot\sin(\beta\cdot h)) + Q_0
\end{aligned}
$$

$$\begin{aligned}
x_1 &\rightarrow \exp(\alpha\cdot h)\cdot((x_1-Q_3)\cdot\cos(\beta\cdot h)+(y_1-Q_3)\cdot\sin(\beta\cdot h))+Q_3\\
y_1 &\rightarrow \exp(\alpha\cdot h)\cdot((y_1-Q_3)\cdot\cos(\beta\cdot h)-(x_1-Q_3)\cdot\sin(\beta\cdot h))+Q_3\\
z_1 &\rightarrow \exp(\alpha\cdot h)\cdot((z_1-Q_4)\cdot\cos(\beta\cdot h)+(w_1-Q_4)\cdot\sin(\beta\cdot h))+Q_4\\
w_1 &\rightarrow \exp(\alpha\cdot h)\cdot((w_1-Q_4)\cdot\cos(\beta\cdot h)-(z_1-Q_4)\cdot\sin(\beta\cdot h))+Q_4
\end{aligned}$$

Figure (2) is KI-ID

FIGURE 2

Data for Fg.(2), KI-ID, are as follows:

Damping and frequency	$\alpha=-0.1:\beta=0.5$
Step size	$h=0.001$
Number of iterations	N = 1000000
Initial conditions	$x=0:y=1:z=0:w=1.5$

Iteration equations for KI-ID are as follows:

$$\begin{aligned}
Q_0 &= Q(x-5.23\cdot w\cdot z)\\
Q_1 &= Q(y-0.1\cdot x)\\
Q(v) &= 5.0\cdot(1-\exp((1-\exp(v))/5.0)))\\
x &\rightarrow \exp(\alpha\cdot h)\cdot((x-Q_1)\cdot\cos(\beta\cdot h)+(y-Q_1)\cdot\sin(\beta\cdot h))+Q_1\\
y &\rightarrow \exp(\alpha\cdot h)\cdot((y-Q_1)\cdot\cos(\beta\cdot h)-(x-Q_1)\cdot\sin(\beta\cdot h))+Q_1\\
z &\rightarrow \exp(\alpha\cdot h)\cdot((z-Q_0)\cdot\cos(\beta\cdot h)+(w-Q_0)\cdot\sin(\beta\cdot h))+Q_0\\
w &\rightarrow \exp(\alpha\cdot h)\cdot((w-Q_0)\cdot\cos(\beta\cdot h)-(z-Q_0)\cdot\sin(\beta\cdot h))+Q_0
\end{aligned}$$

6 The Wave $\Psi(\mathbf{X})$ for any K model may arise from a partial differential equations that must be derived from experiment

Various wave models are possible however any model should "make sense" for possible fluid/wave phenomena in the brain. As such we would expect the PDE to have some relationship to the K models. The rationale for this is that neurons have known dynamics and the dynamics of the wave should be morphologically [4] equivalent to the known sigmoid dynamics. We mention two.

For simplicity of discussion and only focus on the concepts, we examine the two-dimensional case where $\Psi(\mathbf{X})=(Q\circ f)(x,y))$. Let $G=(Q\circ f)$. Define $\Psi(x,y)=G(x+iy)+G(x-iy)$. By a direct computation we have

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}=0$$

Now note that an ID is a function of $x+iy,\quad x-iy$ by definition.

If this is generalized to higher space dimensions, we have

$$\sum_i\frac{\partial^2\Psi}{\partial y_i^2}=0$$

Also, if we have $\Psi(x,y) = G(i(x+y)) + G(-i(x+y))$, then $\Psi_{xx} - \Psi_{yy} = 0$ and so the forcing term in the KIII model, Eq.(17), may be viewed as a wave/field. These two formulations suggest that the field equation we are seeking is a partial differential equation of Ψ using Eq.(17) as a guide in conjunction with experimental data and the theory of mesoscopic brain dynamics.

While the sigmoid function is known to describe neuron binary dynamics, the complex summation of sigmoid functions could be replaced by a morphologically equivalent function which is known to satisfy a wave equation, for example $\sin(u) + \sin(3 \cdot u)/3 + \cdots$. Figure 3 compares using a wave-sigmoid dynamic to a Global wave dynamic in the KII-ID model.

In the KIII-ID model, the function $Q(v)$ which represents the transfer from a wave to an impulse is replaced by a new function that collectively describes the local dynamics without consideringteh specific dynamics of wave-pulse interaction. This is a mathematical abstraction and simplification to is a break from the physics and a transition to just to the collective dynamics of all forces and interactions combined. Making this abstraction alleviates the researcher unskilled in neuroscience from fully understanding the particulars of the wave to pulse dynamic and only considering to mathematical dynamics. While this does place the engineer a step away from the neuroscience, it may also facilitate formulations that will encompass additional insights and provide access to the KIII theory by scientists and engineers unskilled in the details of neuroscience.

FIGURE 3

Data for Fg.(3), KIII-ID, are as follows:

Damping and frequency	$\alpha = -0.03 : \beta = 0.5$
Step size	$h = 0.01$
Number of iterations	N = 1000000
Initial conditions	$x = 0 : y = 0.02 : z = 0 : w = 0.05$
Initial conditions	$x_1 = 0 : y_1 = 0.2 : z_1 = 0 : w_1 = 0.5$
Initial conditions	$x_2 = 0 : y_2 = 0.2 : z_2 = 0 : w_2 = 0.5$

Iteration equations for KIII-ID are as follows:

$$
\begin{aligned}
Q_0 &= Q(w) + Q(x_2 + \Psi(x_2, w_1, z, w) \\
Q_1 &= Q(y_1 + w_1) - Q(z) \\
Q_3 &= Q(y - w) + 1.1 \cdot Q(z) \\
Q_4 &= Q(y - x) \\
Q(v) &= (1 - \exp((1 - \exp(v)))) \quad \text{Plate A} \\
Q(v) &= \sin(v) + \sin(3 \cdot v)/3 \quad \text{Plate B} \\
\Psi(x,y,z,w) &= \exp(\alpha \cdot x) \cdot \cos(\alpha \cdot y) + \sin(5 \cdot z) \cdot \cos(\cos(x) \cdot w) \\
x &\to \exp(\alpha \cdot h) \cdot ((x - Q_1) \cdot \cos(\beta \cdot h) + (y - Q_1) \cdot \sin(\beta \cdot h)) + Q_1 \\
y &\to \exp(\alpha \cdot h) \cdot ((y - Q_1) \cdot \cos(\beta \cdot h) - (x - Q_1) \cdot \sin(\beta \cdot h)) + Q_1
\end{aligned}
$$

$$
\begin{aligned}
z &\to \exp(\alpha \cdot h) \cdot ((z - Q_0) \cdot \cos(\beta \cdot h) + (w - Q_0) \cdot \sin(\beta \cdot h)) + Q_0 \\
w &\to \exp(\alpha \cdot h) \cdot ((w - Q_0) \cdot \cos(\beta \cdot h) - (z - Q_0) \cdot \sin(\beta \cdot h)) + Q_0 \\
x_1 &\to \exp(\alpha \cdot h) \cdot ((x_1 - Q_3) \cdot \cos(\beta \cdot h) + (y_1 - Q_3) \cdot \sin(\beta \cdot h)) + Q_3 \\
y_1 &\to \exp(\alpha \cdot h) \cdot ((y_1 - Q_3) \cdot \cos(\beta \cdot h) - (x_1 - Q_3) \cdot \sin(\beta \cdot h)) + Q_3 \\
z_1 &\to \exp(\alpha \cdot h) \cdot ((z_1 - Q_2) \cdot \cos(\beta \cdot h) + (w_1 - Q_2) \cdot \sin(\beta \cdot h)) + Q_2 \\
w_1 &\to \exp(\alpha \cdot h) \cdot ((w_1 - Q_2) \cdot \cos(\beta \cdot h) - (z_1 - Q_2) \cdot \sin(\beta \cdot h)) + Q_2 \\
x_2 &\to \exp(\alpha \cdot h) \cdot ((x_2 - Q_5) \cdot \cos(\beta \cdot h) + (y_2 - Q_5) \cdot \sin(\beta \cdot h)) + Q_5 \\
y_2 &\to \exp(\alpha \cdot h) \cdot ((y_2 - Q_5) \cdot \cos(\beta \cdot h) - (x_2 - Q_5) \cdot \sin(\beta \cdot h)) + Q_5 \\
z_2 &\to \exp(\alpha \cdot h) \cdot ((z_2 - Q_4) \cdot \cos(\beta \cdot h) + (w_2 - Q_4) \cdot \sin(\beta \cdot h)) + Q_4 \\
w_2 &\to \exp(\alpha \cdot h) \cdot ((w_2 - Q_4) \cdot \cos(\beta \cdot h) - (z_2 - Q_4) \cdot \sin(\beta \cdot h)) + Q_4
\end{aligned}
$$

7 Summary

Starting with the KIII wave theory of Freeman-Kozma, we derived a top-down mathematical model, KIII-ID, which used the concept of stretching and folding in place of the laws of physics. We noted that the ID model has a mathematical foundation that has broad applicability to many dynamical systems including the KIII ODEs. We discussed some of the simplifying advantages of the KIII-ID approach and then we used the KII-ID model to morphologically replicate results from the KIII model of known EEGs. Finally we suggested that the sigmoid function could be replaced by solutions of wave equations which may lead to further simplifications of the KIII theory and make it more accessible to researchers without an extensive knowledge of neurodynamics as well as more amenable to formal scientific proof.

8 References

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[2] Brown, R. [2014] "ISF Part I" to appear

[3] Freeman, W.J. [2000] *Neurodynamics: An Exploration in Mesoscopic Brain dynamics*, Springer, New York

[4] Smale S., [1967] Differentiable Dynamical Systems. Bull Am Math Soc **73** 747-817.

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List of Figures

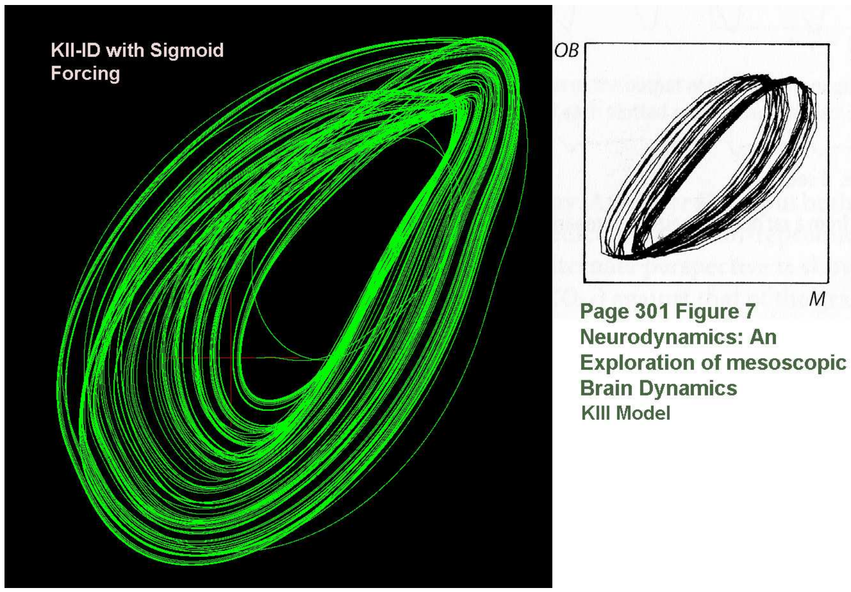

Figure 1: Page 301, figure 7, Left Plate

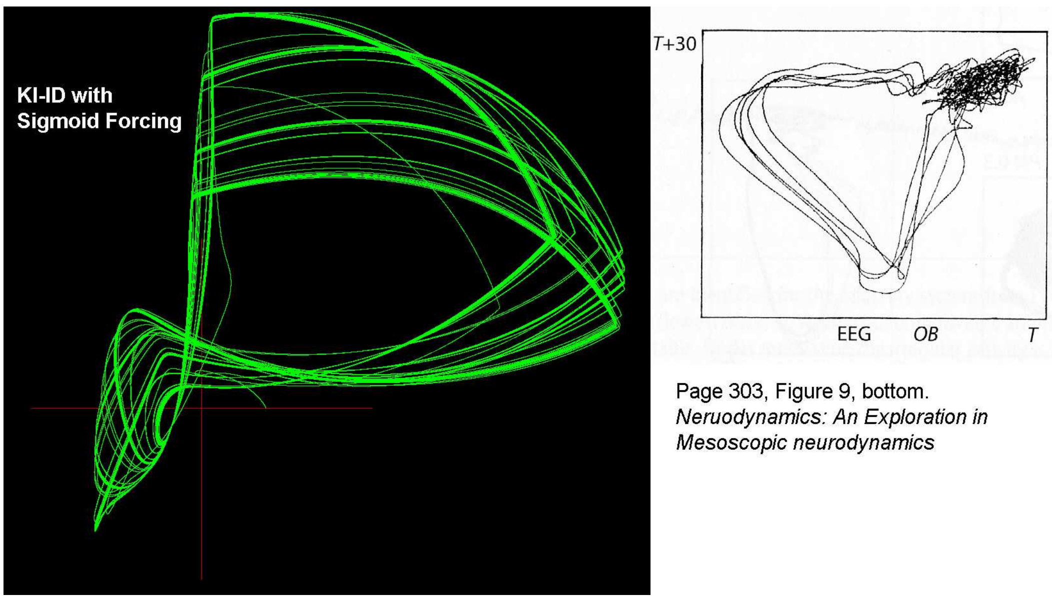

Figure 2: Page 303, figure 9, bottom Plate

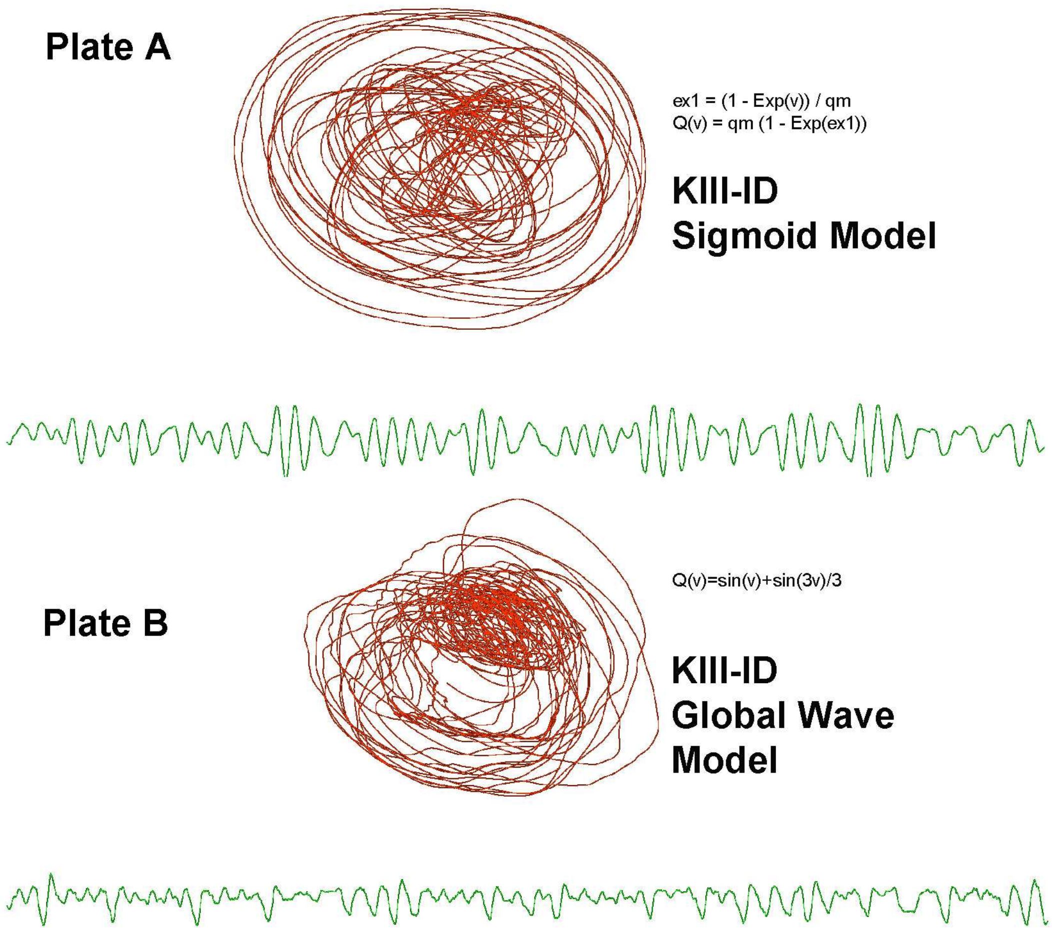

Figure 3: KIII-ID Sigmoid Model versus the KIII-ID Wave Model